Persistence of Topological Phases in Non-Hermitian Quantum Walks

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Abstract

Discrete-time quantum walks (DTQWs) are known to exhibit exotic topological states and phases. Physical realization of quantum walks in a noisy environment may destroy these phases. We investigate the behavior of topological states in quantum walks in the presence of a lossy environment. The environmental effects in the quantum walk dynamics are addressed using the non-Hermitian Hamiltonian approach. We show that the topological phases of the quantum walks are robust against moderate losses. The topological order in one-dimensional (1D) split-step quantum walk persists as long as the Hamiltonian is $\mathcal{PT}$-symmetric. Although the topological nature persists in two-dimensional (2D) quantum walks as well, the $\mathcal{PT}$-symmetry has no role to play there. Furthermore, we observe the noise-induced topological phase transition in two-dimensional quantum walks.

1D DTQW

- A DTQW consists of a quantum walker over a 1D lattice whose evolution consists of two operation spin flip operator $R(\theta)$ and conditional translational operator T such that time evolution operation reads

$$U(\theta) = TR(\theta) \quad (1)$$

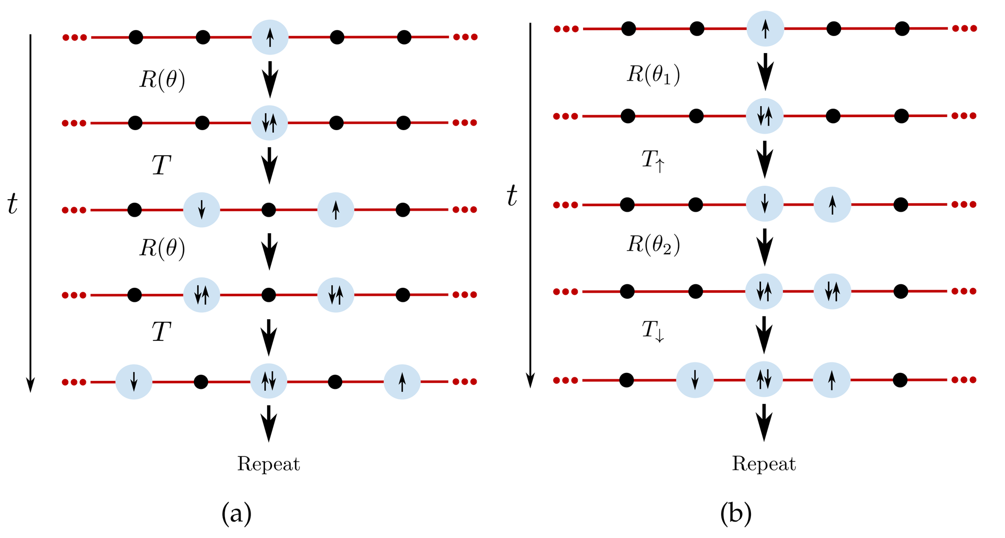

Figure 1: Schematic protocol for (a) 1D DTQW and (b) for 1D SSQW.

- A more enriched class of 1D DTQW is split-step quantum walk (SSQW), which involves splitting the conditional shift operator T into left-shift ($T_\downarrow$) and right-shift ($T_\uparrow$) operators, separated by an additional coin toss $R(\theta_2)$ [2] such that

$$U_{\text{SS}}(\theta_1, \theta_2) = T_\downarrow R(\theta_2) T_\uparrow R(\theta_1)$$

2D DTQW

- A 2D DTQW on a triangular lattice consists of three conditional shift operators separated by coin-flip operations such that

$$U_{2D}(\theta_1, \theta_2) = T_{xy} R(\theta_1) T_y R(\theta_1) T_y R(\theta_2) T_x R(\theta_1) T_x.$$

- An equivalent 2D DTQW on a square lattice and using cyclic property we can rewrite time evolution operator given as

$$U_{2D}(\theta_1, \theta_2) = T_y R(\theta_1) T_y R(\theta_2) T_x R(\theta_1) T_x.$$

Non-Hermitian QW

- We extend 1D SSQW to non-hermitian domain by introducing scaling operator $G = e^{\gamma\sigma_z}$ [4]

$$U_{\text{SS}}^{\text{NU}}(\theta_1, \theta_2, \gamma) = T_\downarrow G_\gamma R(\theta_2) T_\uparrow G_\gamma^{-1} R(\theta_1).$$

- We go to momentum (quasi) basis by performing Fourier transform and write corresponding generator, $H_{\text{NU}}(\theta_1, \theta_2, \gamma)$ [2] which reads

$$H_{\text{NU}}(\theta_1, \theta_2, \gamma) = \bigoplus_k E(k)\, \hat{\mathbf{n}}(k) \cdot \sigma,$$

with quasi-energy $E(k)$ and Bloch vector $\hat{\mathbf{n}}(k)$.

- For $\gamma \neq 0$, G_γ as well as U becomes non-unitary and the corresponding generator non-hermitian.
- In 2D, we introduce loss and gain in x as well as y axis which results in non-hermitian dynamics such that

$$U_{2D}^{\text{NU}}(\theta_1, \theta_2, \gamma) = G_{\gamma_y} T_y R(\theta_1) G_{\gamma_y}^{-1} T_y R(\theta_2) G_{\gamma_x} T_x R(\theta_1) G_{\gamma_x}^{-1} T_x.$$

Topological Phases in QWs

- A quantum system can support various topological phases on the basis of the symmetries possessed by it's respective Hamiltonian.

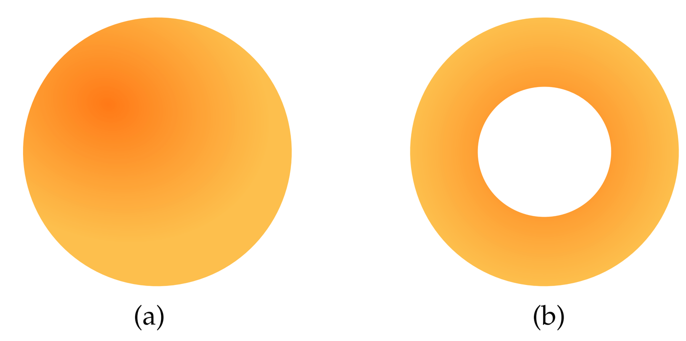

Figure 2: A *laddoo* (a) and an *emarti* (b) have genus 0 and 1 respectively and hence topological different objects.

- 1D SSQW is known to exhibit topological phases which are characterized using winding number, $W = 0, 1$.
- In 2D DTQW topological phases are characterized by Chern number C. 2D DTQW supports topological phases with $C = \pm 1, 0$.

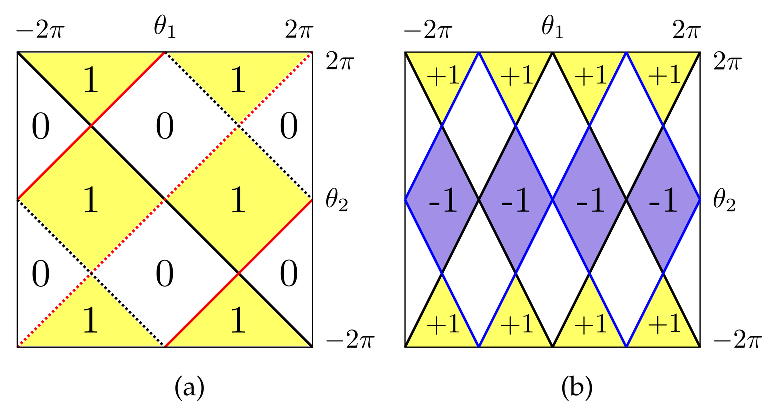

Figure 3: (Color online) Different topological phases realized in (a) 1D SSQW and (b) 2D DTQW as a function of θ_1 and θ_2.

Results

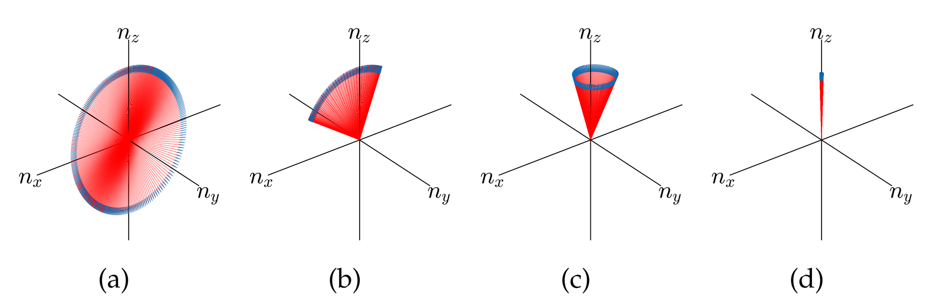

Figure 4: (Color online) Winding of the Bloch vector around the origin with the lattice size, $N = 100$ (a) $\theta_1 = -3\pi/8$, $\theta_2 = \pi/8$, $\gamma = 0.25$ (b) $\theta_1 = -3\pi/8$, $\theta_2 = 5\pi/8$, $\gamma = 0.25$ (c) $\theta_1 = -3\pi/8$, $\theta_2 = \pi/8$, $\gamma = 1.8$ (d) $\theta_1 = -3\pi/8$, $\theta_2 = \pi/8$, $\gamma = 3.0$.

- In 1D SSQW, the topological phases are unaffected even when the system is non-Hermitian [1] (i.e., $\gamma \neq 0$), as far as the system possesses a real spectrum following the $\mathcal{PT}$-symmetry [5].
- The topological nature of the system vanishes asymptotically as we cross the exceptional point γ_c, which can be seen in Fig. 5(a), 5(b).
- In 2D DTQW, the persistence of the Chern number has been observed C as well until the scaling factor γ reaches a critical value.
- We cannot associate any symmetry breaking with the point where the topological phase transition happens due to the absence of the symmetry in 2D DTQW.
- In 2D DTQW we observe sharp transition and it further shows noise-induced topological phase.

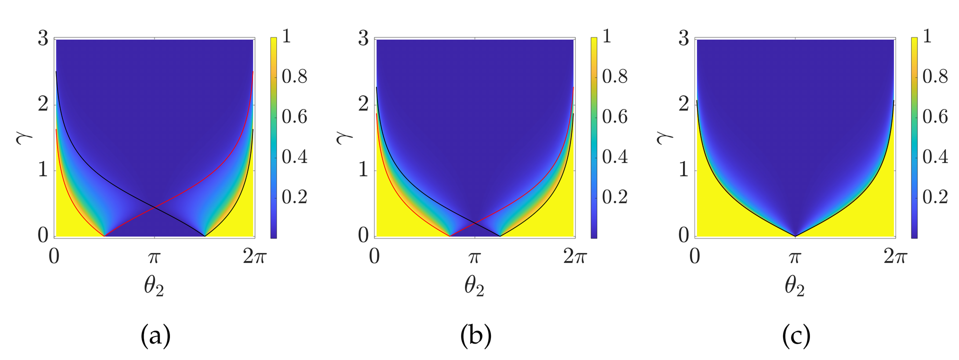

Figure 5: (Color online) Winding number for lower energy band W_- as a function of γ and θ_2, and (a) $\theta_1 = -\pi/2$ (b) $\theta_1 = -3\pi/4$ (c) $\theta_1 = -\pi$. The system size is taken to be $N = 100$. The red and black lines in all of the panels represent γ_c for $(k, E) = (0, 0)$ and $(k, E) = (\pi, 0)$, respectively.

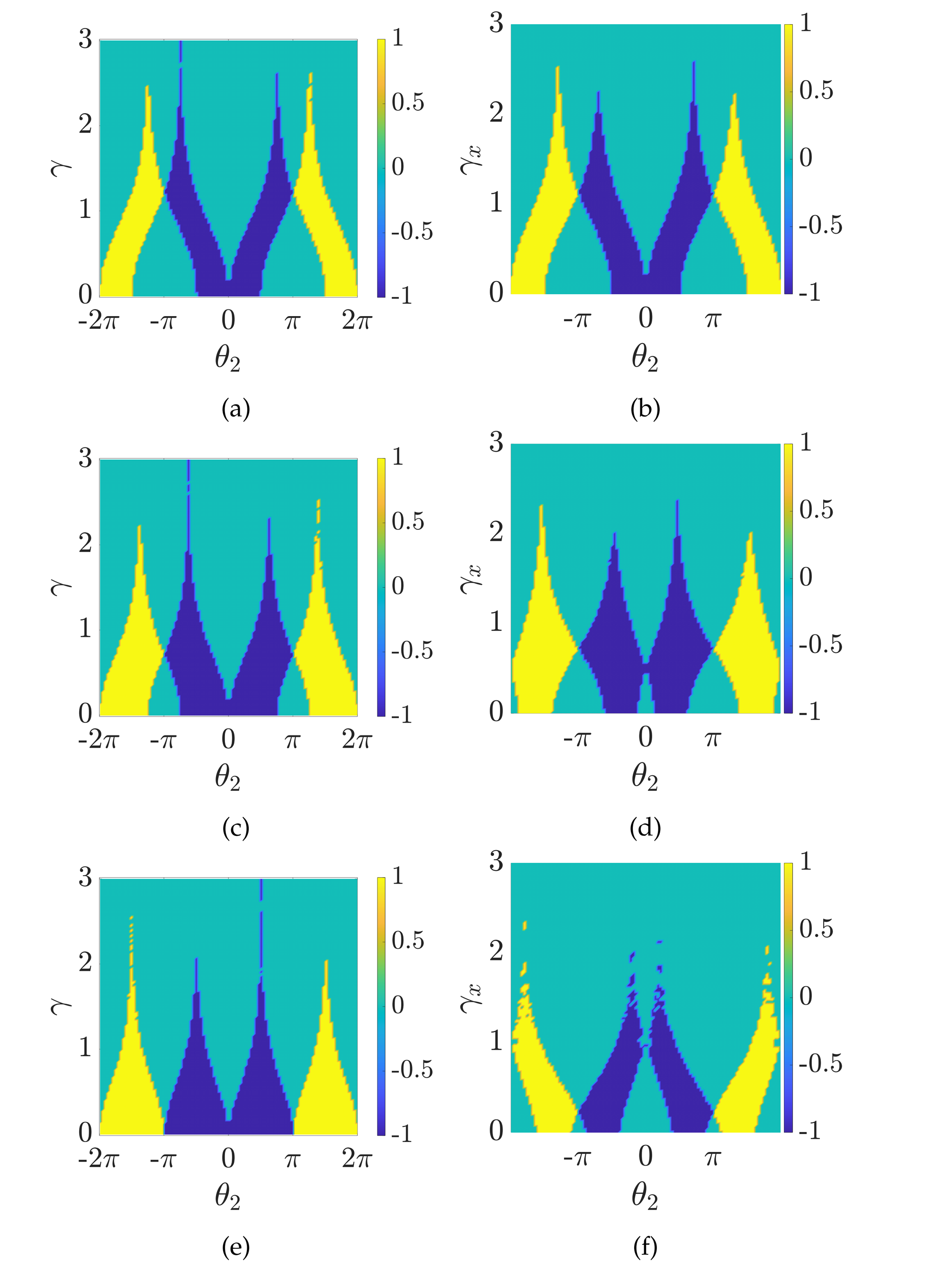

Figure 6: (Color online) Effect of γ_x on Chern number is plotted with varying θ_2 for $\gamma_y = 0$ (a) $\theta_1 = \pi/4$ (c) $\theta_1 = 3\pi/8$ (e) $\theta_1 = 3\pi/2$. In the right column, (b) $\gamma_y = 0.1$, (d) $\gamma_y = 0.5$, (f) $\gamma_y = 1.0$ respectively. The system size is taken to be $N = 100$.

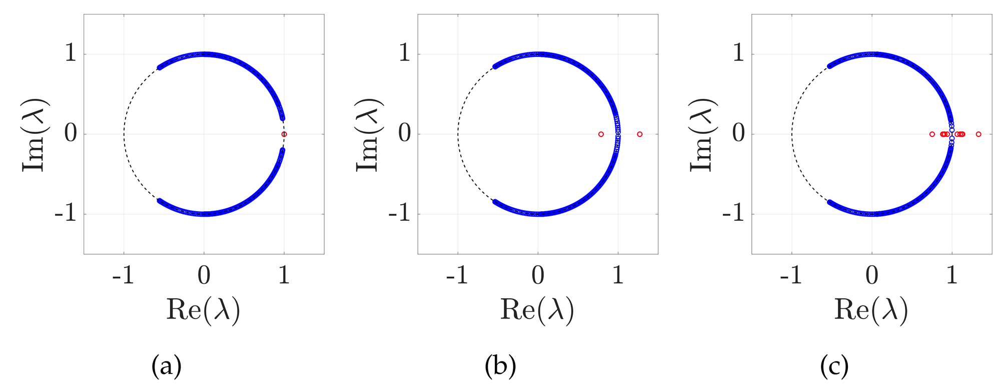

Figure 7: (Color online) The eigenvalues, λ of the time evolution operator in Eq. () are plotted for $(\theta_1^1, \theta_2^1) = (-3\pi/8, \pi/4)$ and $(\theta_1^0, \theta_2^0) = (-3\pi/8, 5\pi/8)$ and different values of γ. In (a) $\gamma = 0$, (b) $\gamma = \min(\gamma^1, \gamma^2) = 0.2110$, and (c) $\gamma = 0.25$.

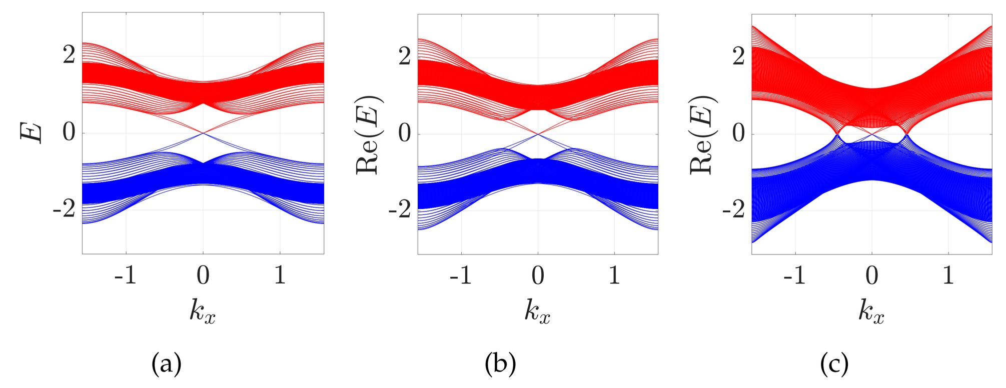

Figure 8: (Color online) Energy bands for the 2D DTQW are plotted for inhomogeneous lattice with lattice size 201×201. We have chosen $(\theta_1^0, \theta_2^0) = (3\pi/2, 2\pi/2)$ and $(\theta_1^{+1}, \theta_2^{+1}) = (7\pi/6, 7\pi/6)$ which correspond to $C = 0$ and $C = +1$, respectively, for the two parts of the lattice. The scaling parameters are chosen to be $\gamma_x = \gamma_y = 0,\ 0.3,\ 0.45$ for (a), (b) and (c), respectively.

Conclusion

- The topological phases of the quantum walks are robust against moderate losses.
- The topological order in 1D SSQW persists as long as the Hamiltonian is $\mathcal{PT}$ – symmetry.
- The topological nature persists in 2D DTQW as well, although, $\mathcal{PT}$ – symmetry does not play any role there.
- We observe noise-induced topological phase transition in 2D DTQW, which was absent in 1D systems.
- We also observe the robustness nature of edge states with scaling.

References

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